8.1 - Defining and Using Sequences and Series

Warmup

Find the indicated term in each arithmetic sequence.

1. a_{12} for $-17, -13, -9, \ldots$

   27

2. a_{21} for $10, 7, 4, \ldots$

   -50

3. a_{32} for $4, 7, 10, 13, \ldots$

   97

4. a_{10} for $8, 3, -2, \ldots$

   -37

5. a_{12} for $\frac{3}{4}, \frac{3}{2}, \frac{9}{4}, \ldots$

   9

6. a_{10} for $\frac{5}{6}, \frac{7}{6}, \frac{3}{2}, \ldots$

   $\frac{23}{6}$

8.1 - Defining and Using Sequences and Series

Sigma Notation

Arithmetic Series

n ends at 5

$$\sum_{n=1}^{5} 3n$$

n starts at 1

This is read as:
the summation from 1 to 5 of 3n

$$\sum_{n=1}^{5} 3n = 3(1) + 3(2) + 3(3) + 3(4) + 3(5) = 45$$

8.1 - Defining and Using Sequences and Series

Sigma Notation

Geometric Series

n ends at 5

$$\sum_{j=1}^{5} 2(4)^{j-1}$$

n starts at 1

$$\sum_{j=1}^{5} 2(4)^{j-1} = 2(4)^0 + 2(4)^1 + 2(4)^2 + 2(4)^3 + 2(4)^4 = 682$$

8.1 - Defining and Using Sequences and Series

Sigma Notation

Geometric Series

$$\sum_{j=1}^{5} 2(4)^{j-1} = 2(4)^0 + 2(4)^1 + 2(4)^2 + 2(4)^3 + 2(4)^4 = 682$$

Practice - expand and find the sum

1. $\sum_{n=4}^{8} 4^n$

   $87{,}296$

2. $\sum_{n=1}^{4} 24\left(-\frac{1}{2}\right)^n$

   $-\frac{15}{2}$

8.1 - Defining and Using Sequences and Series

Sigma Notation

$$\sum_{j=1}^{5} ?? = 5 + 9 + 13 + 17 + 21$$

Arithmetic or geometric?

$$= 5 + (5 + 4) + (5 + 2 \cdot 4) + (5 + 3 \cdot 4) + (5 + 4 \cdot 4)$$

$$= \sum_{n=1}^{5} (4n + 1)$$

Practice - write in sigma notation

1. $7 + 10 + 13 + 16 + 19$

$$\sum_{n=1}^{5} (7 + 3(n - 1))$$

$$= \sum_{n=1}^{5} (3n + 4)$$

2. $15 + 11 + 7 + 3 + (-1)$

$$\sum_{n=1}^{5} (15 + (-4)(n - 1))$$

$$= \sum_{n=1}^{5} (-4n + 19)$$

8.1 - Defining and Using Sequences and Series

Sigma Notation

$$2 + (-2i) + (-2) + 2i$$

$$\sum_{n=1}^{4} 2(-i)^{n-1}$$

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Series Equations

8.1 - Defining and Using Sequences and Series

Warmup

Find the next two terms for each geometric sequence.

1. $2, 6, 18, \ldots$

   $54, 162$

2. $729, 243, 81, \ldots$

   $27, 9$

3. $20, 30, 45, \ldots$

   $67.5, 101.25$

4. $90, 30, 10, \ldots$

   $\frac{10}{3}, \frac{10}{9}$

5. $\frac{1}{27}, \frac{1}{9}, \frac{1}{3}, \ldots$

   $1, 3$

6. $-\frac{1}{4}, \frac{1}{2}, -1, \ldots$

   $2, -4$

8.1 - Defining and Using Sequences and Series

Formulas for Special Series

Sum of *n* terms of 1: $\sum_{i=1}^{n} 1 = n$

Sum of first *n* positive integers: $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Sum of squares of first *n* positive integers: $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

Find the sum.

1. $\sum_{i=1}^{9} 3$ 27

2. $\sum_{i=1}^{5} 8i$ 120

3. $\sum_{k=3}^{7} (k^2 - 1)$ 130

8.2 - Analyzing Arithmetic Sequences and Series

Adding the terms of an arithmetic sequence is called an **arithmetic series**. The sum of the first n terms of an arithmetic series is denoted by S_n .

You can develop the rule for S_n as follows:

$$\begin{array}{rl} S_n = a_1 & + (a_1 + d) + (a_1 + 2d) + \cdots + a_n \\ S_n = a_n & + (a_n - d) + (a_n - 2d) + \cdots + a_1 \\ \hline 2S_n = & \underbrace{(a_1 + a_n) + (a_1 + a_n) + (a_1 + a_n) + \cdots + (a_1 + a_n)}_{(a_1 + a_n) \text{ is added } n \text{ times.}} \end{array}$$

You can conclude that $2S_n = n(a_1 + a_n)$, which leads to the following result.

8.2 - Analyzing Arithmetic Sequences and Series

The Sum of a Finite Arithmetic Series

The sum of the first n terms of an arithmetic series is

$$S_n = n\left(\frac{a_1 + a_n}{2}\right).$$

In words, S_n is the mean of the first and nth terms, multiplied by the number of terms.

Find the sum:

$$\sum_{k=1}^{12} (7k + 2)$$

570

8.2 - Analyzing Arithmetic Sequences and Series

The Sum of a Finite Arithmetic Series

The sum of the first n terms of an arithmetic series is

$$S_n = n\left(\frac{a_1 + a_n}{2}\right).$$

In words, S_n is the mean of the first and nth terms, multiplied by the number of terms.

1. Find S_n

$a_1 = 5$

$d = 12$

$n = 7$

$S_7 = 287$

2. Find S_n

$a_1 = 9$

$d = -6$

$n = 14$

$S_{14} = -420$

8.2 - Analyzing Arithmetic Sequences and Series

Sum of first 20 positive integers divisible by 3.

$$a_n = 3 + (n-1)3 = 3n$$

$$\sum_{n=1}^{20} 3n = 20\left(\frac{60+3}{2}\right) = 630$$

8.3 - Analyzing Geometric Sequences and Series

Adding the terms of a geometric sequence is called a **geometric series**. The sum of the first n terms of a geometric series is denoted by S_n .

You can develop the rule for S_n as follows:

$$\begin{array}{rcl} S_n &=& a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} \\ -rS_n &=& \quad - a_1 r - a_1 r^2 - a_1 r^3 - \cdots - a_1 r^{n-1} - a_1 r^n \\ \hline S_n - rS_n &=& a_1 + 0 + 0 + 0 + \cdots + 0 \quad - a_1 r^n \end{array}$$

$$S_n(1 - r) = a_1(1 - r^n)$$

8.3 - Analyzing Geometric Sequences and Series

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right).$$

1. $\sum_{k=1}^{12} 6(-2)^{k-1}$

-8190

2. Find S_6

$a_1 = 3$

$r = -2$

$S_6 = -63$

3. Find S_5

$a_1 = 7$

$r = 3$

$S_5 = 847$

8.3 - Analyzing Geometric Sequences and Series

Practice:

1. Arithmetic n=10, $a_1 = 3$, $a_{10} = 300$

1515

2. S_{20} for $3 + 11 + 19 + 27 + \ldots$

1580

3. $50 + 33 + 16 + \ldots + (-86)$

-162

4. Geometric n=10, $r = 2$, $a_1 = \frac{1}{4}$

$\frac{1023}{4} = 255.75$

5. S_8 for $48 - 24 + 12 - 6 + \ldots$

31.875

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